A

Surname _____

Other Names _____

Centre Number _____

Candidate Number _____

Candidate Signature _____

I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 1

7367/1

Friday 22 May 2020 Morning

Time allowed: 2 hours

At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.

[Turn over]

JUN207367101

For this paper you must have:

- **an AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.**
- **You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)**

INSTRUCTIONS

- **Use black ink or black ball-point pen. Pencil should only be used for drawing.**
- **Answer ALL questions.**
- **You must answer each question in the space provided for that question.**
- **Do NOT write on blank pages.**
- **Show all necessary working; otherwise marks for method may be lost.**
- **Do all rough work in this book. Cross through any work you do not want to be marked.**

INFORMATION

- **The marks for questions are shown in brackets.**
- **The maximum mark for this paper is 100.**

ADVICE

- **Unless stated otherwise, you may quote formulae, without proof, from the booklet.**
- **You do not necessarily need to use all the space provided.**

DO NOT TURN OVER UNTIL TOLD TO DO SO

Answer ALL questions in the space provided.

1 **Which of the integrals below is NOT an improper integral?**

Circle your answer. [1 mark]

$$\int_0^{\infty} e^{-x}\,dx$$

$$\int_0^{2} \frac{1}{1-x^2}\,dx$$

$$\int_0^{1} \sqrt{x}\,dx$$

$$\int_0^{1} \frac{1}{\sqrt{x}}\,dx$$

2 Which one of the matrices below represents a rotation of 90^{o} about the x-axis?

Circle your answer. [1 mark]

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

3 The quadratic equation $ax^2 + bx + c = 0$ $(a, b, c \in \mathbb{R})$ has real roots α and β.

One of the four statements below is incorrect.

Which statement is INCORRECT?

Tick (✓) ONE box. [1 mark]

☐ $c = 0 \Rightarrow \alpha = 0$ or $\beta = 0$

☐ $c = a \Rightarrow \alpha$ is the reciprocal of β

☐ $b < 0$ and $c < 0 \Rightarrow \alpha > 0$ and $\beta > 0$

☐ $b = 0 \Rightarrow \alpha = -\beta$

4 It is given that $1 - 3\mathrm{i}$ is one root of the quartic equation

$$z^4 - 2z^3 + pz^2 + rz + 80 = 0$$

where p and r are real numbers.

4 (a) Express $z^4 - 2z^3 + pz^2 + rz + 80$ as the product of two quadratic factors with real coefficients. [4 marks]

[Turn over]

4(b) **Find the value of p and the value of r. [2 marks]**

BLANK PAGE

[Turn over]

5 H_1 is the locus of points such that the distance from the point (5, 0) is twice the distance from the line $x = 2$

5 (a) Show that the equation of H_1 can be written in the form

$$(x-1)^2 - \frac{y^2}{q} = r$$

where q and r are integers.
[5 marks]

[Turn over]

5(b) H_2 is the hyperbola

$$x^2 - y^2 = 4$$

Describe fully a sequence of two transformations which maps the graph of H_2 onto the graph of H_1
[4 marks]

[Turn over]

6 Let w be the root of the equation $z^7 = 1$ that has the smallest argument α in the interval $0 < \alpha < \pi$

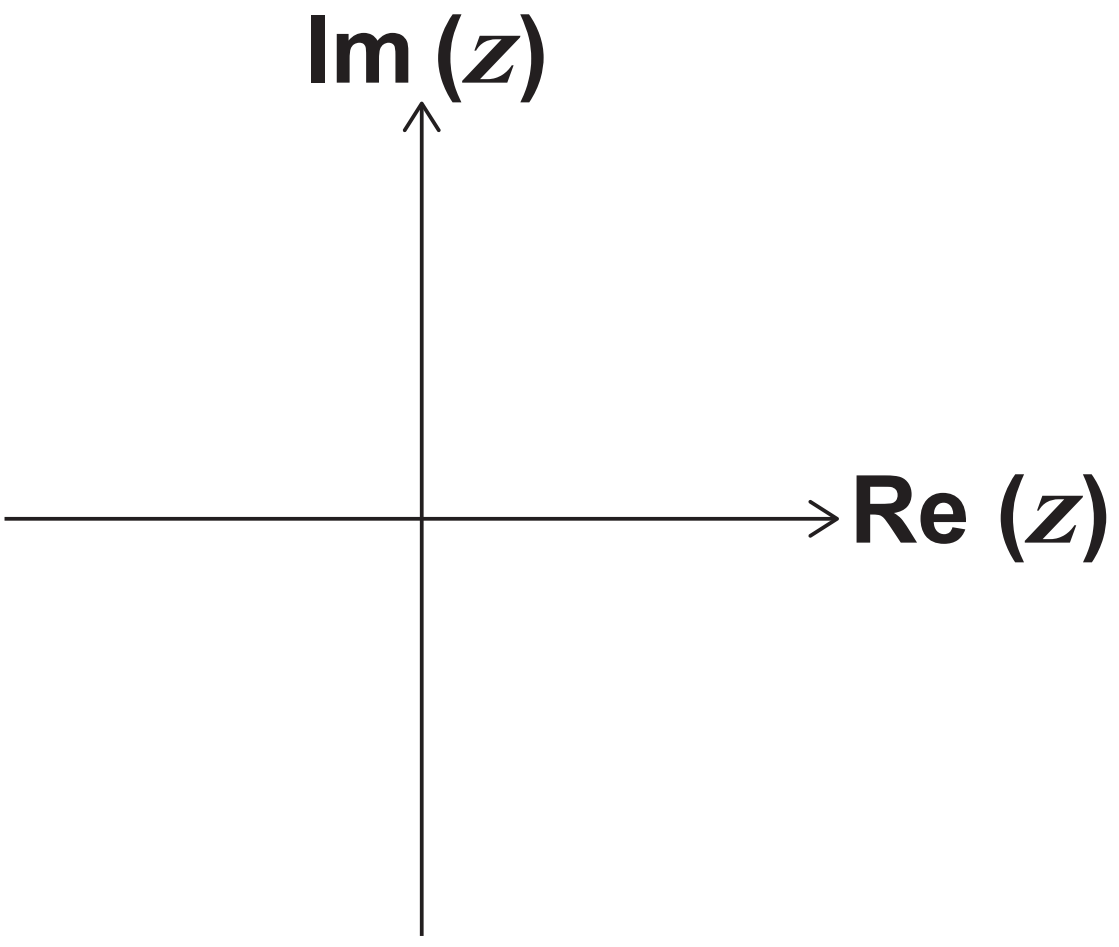
6 (a) Prove that w^n is also a root of the equation $z^7 = 1$ for any integer n. [1 mark]

6(b) Prove that

$$1 + w + w^2 + w^3 + w^4 + w^5 + w^6 = 0$$

[2 marks]

6 (c) Show the positions of w, w^2, w^3, w^4, w^5, and w^6 on the Argand diagram below. [2 marks]



6(d) **Prove that**

$$\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = -\frac{1}{2}$$

[4 marks]

[Turn over]

7 Three planes have equations

$$(4k + 1)x - 3y + (k - 5)z = 3$$

$$(k - 1)x + (3 - k)y + 2z = 1$$

$$7x - 3y + 4z = 2$$

7 (a) The planes do NOT meet at a unique point.

Show that $k = 4.5$ is one possible value of k, and find the other possible value of k.
[3 marks]

[Turn over]

7(b) For each value of k found in part (a), identify the configuration of the given planes.

In each case fully justify your answer, stating whether or not the equations of the planes form a consistent system. [4 marks]

[Turn over]

8 The three roots of the equation

$$4x^3 - 12x^2 - 13x + k = 0$$

where k is a constant, form an arithmetic sequence.

Find the roots of the equation.
[6 marks]

[Turn over]

[Turn over]

9 The function f is defined by

$$\mathrm{f}(x) = \frac{x(x+3)}{x+4} \qquad (x \in \mathbb{R},\ x \neq -4)$$

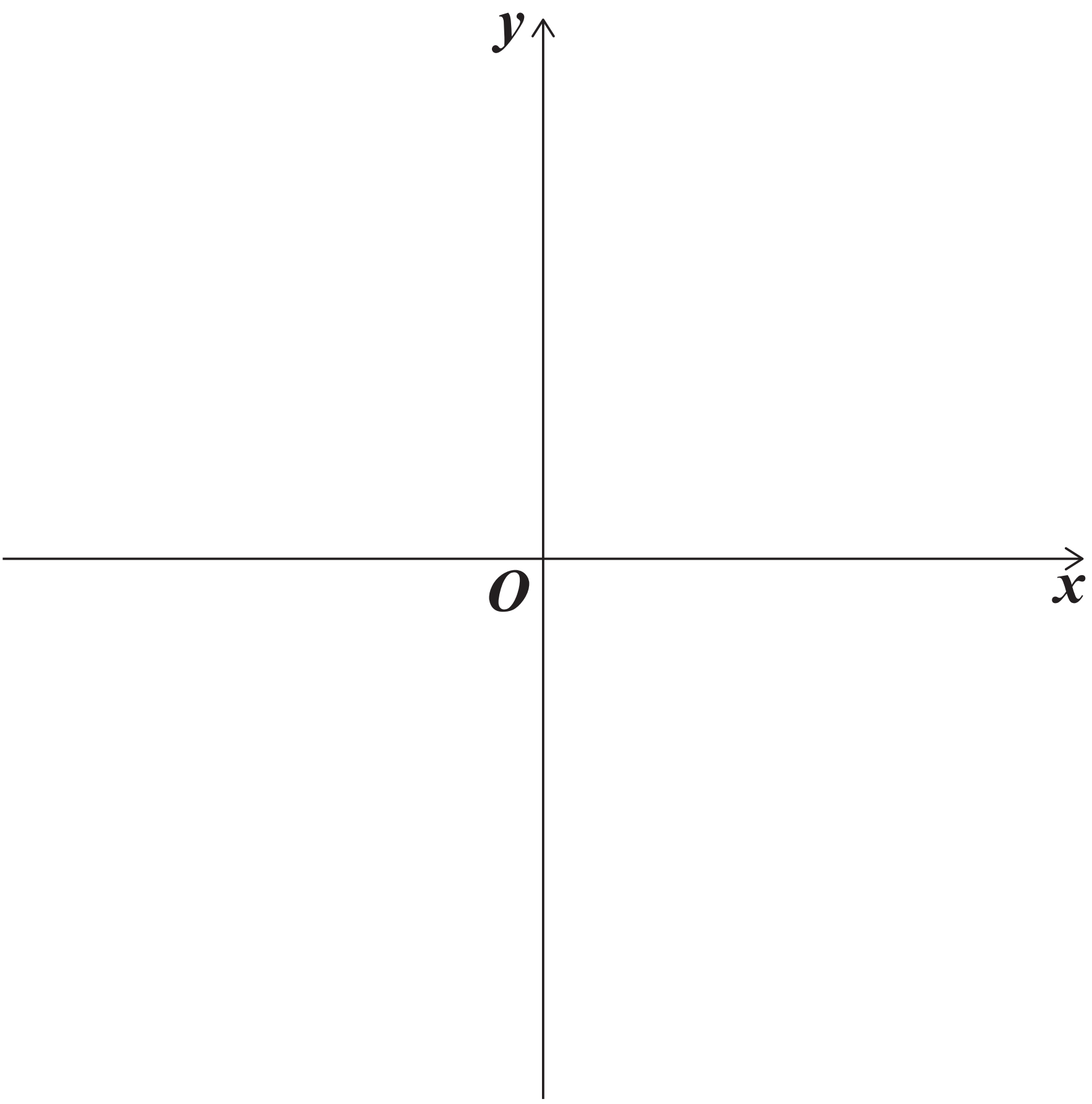
9 (a) Find the interval (a, b) in which $\mathrm{f}(x)$ does not take any values.
Fully justify your answer.
[5 marks]

[Turn over]

9(b) Find the coordinates of the two stationary points of the graph of $y = \mathrm{f}(x)$ [2 marks]

9 (c) Show that the graph of $y = \mathrm{f}(x)$ has an oblique asymptote and find its equation. [2 marks]

9(d) Sketch the graph of $y = \mathrm{f}(x)$ on the axes below. [4 marks]



10 (a) Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2y}{x} = \frac{x+3}{x(x-1)(x^2+3)} \qquad (x > 1)$$

[8 marks]

[Turn over]

[Turn over]

10(b) Find the particular solution for which $y = 0$ when $x = 3$

Give your answer in the form $y = \mathrm{f}(x)$ [2 marks]

_____

_____

[Turn over]

11 The lines l_1, l_2 and l_3 are defined as follows.

$$l_1 : \left(\mathbf{r} - \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix} \right) \times \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} = 0$$

$$l_2 : \left(\mathbf{r} - \begin{bmatrix} -3 \\ 2 \\ 7 \end{bmatrix} \right) \times \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = 0$$

$$l_3 : \left(\mathbf{r} - \begin{bmatrix} -5 \\ 12 \\ -4 \end{bmatrix} \right) \times \begin{bmatrix} 4 \\ 0 \\ 9 \end{bmatrix} = 0$$

11 (a) (i) Explain how you know that two of the lines are parallel. [1 mark]

11 (a) (ii) Show that the perpendicular distance between these two parallel lines is 7.95 units, correct to three significant figures.
[5 marks]

[Turn over]

50

11(b) Show that the lines l_1 and l_3 meet, and find the coordinates of their point of intersection. [5 marks]

[Turn over]

12 (a) Use the definition of the cosh function to prove that

$$\cosh^{-1}\left(\frac{x}{a}\right) = \ln\left(\frac{x + \sqrt{x^2 - a^2}}{a}\right)$$

for $a > 0$

[6 marks]

[Turn over]

[Turn over]

12 (b) The formulae booklet gives the integral of $\frac{1}{\sqrt{x^2 - a^2}}$ as

$$\cosh^{-1}\left(\frac{x}{a}\right)$$

or $\ln(x + \sqrt{x^2 - a^2}) + c$

Ronald says that this contradicts the result given in part (a).

Explain why Ronald is wrong.
[2 marks]

[Turn over]

13 Two light elastic strings each have one end attached to a particle *B* of mass $3c$ kg, which rests on a smooth horizontal table.

The other ends of the strings are attached to the fixed points *A* and *C*, which are 8 metres apart. *ABC* is a horizontal line.

String *AB* has a natural length of 4 metres and a stiffness of $5c$ newtons per metre.

String *BC* has a natural length of 1 metre and a stiffness of c newtons per metre.

The particle is pulled a distance of $\frac{1}{3}$ metre from its equilibrium position towards *A*, and released from rest.

13 (a) Show that the particle moves with simple harmonic motion.
[8 marks]

[Turn over]

13(b) Find the speed of the particle when it is at a point P, a distance $\frac{1}{4}$ metre from the equilibrium position. Give your answer to two significant figures. [4 marks]

[Turn over]

14 (a) **Given that**

$$\sinh(A + B) =$$

$$\sinh A \cosh B + \cosh A \sinh B$$

express $\sinh(m + 1)x$ **and** $\sinh(m - 1)x$ **in terms of** $\sinh mx$, $\cosh mx$, $\sinh x$ **and** $\cosh x$
[1 mark]

14(b) Hence find the sum of the series

$$C_n = \cosh x + \cosh 2x + \cdots + \cosh nx$$

in terms of $\sinh x$, $\sinh nx$ and $\sinh (n+1)x$ [5 marks]

[Turn over]

END OF QUESTIONS

BLANK PAGE

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
TOTAL	